BAULKHAM HILLS HIGH SCHOOL

PERSEVERE

2019 **HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION**

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen, black is preferred
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 – 17)

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer page in the writing booklet for Questions 1 – 10.

---

1 What is the value of i^{2019} ?

A. i

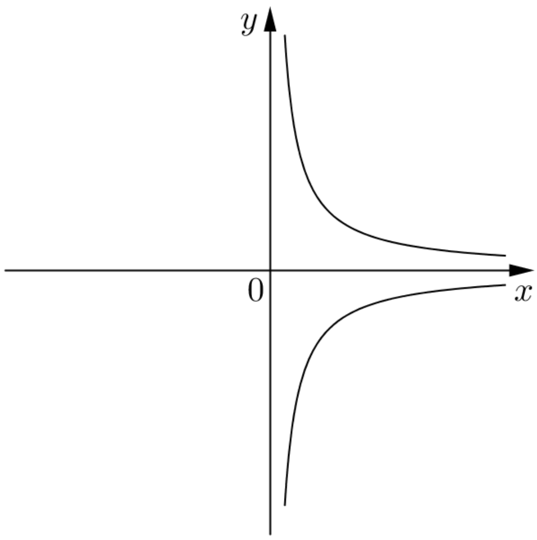
B. $-i$

C. 1

D. –1

---

2 Which of the equations below best represents the following graph?



A. $|x|y = 1$

B. $x|y| = 1$

C. $|xy| = 1$

D. $x^2y^2 = 1$

3 What are the equations of the asymptotes of the hyperbola $4x^2 - 25y^2 = 100$?

A. $y = \pm \frac{2}{5}x$

B. $y = \pm \frac{4}{25}x$

C. $y = \pm \frac{5}{2}x$

D. $y = \pm \frac{25}{4}x$

---

4 Multiplying a non-zero complex number by $\frac{1+i}{1-i}$ results in a rotation about the origin on an Argand diagram. What is the rotation?

A. Clockwise by $\frac{\pi}{2}$ radians

B. Clockwise by $\frac{\pi}{4}$ radians

C. Anticlockwise by $\frac{\pi}{2}$ radians

D. Anticlockwise by $\frac{\pi}{4}$ radians

---

5 The polynomial equation $x^3 - 3x^2 - x + 2 = 0$ has roots α, β and γ. Which of the following polynomial equations has roots $2\alpha + \beta + \gamma$, $\alpha + 2\beta + \gamma$ and $\alpha + \beta + 2\gamma$?

A. $x^3 - 6x^2 - 2x + 4 = 0$

B. $x^3 + 6x^2 + 8x - 1 = 0$

C. $x^3 + 12x^2 - 44x + 49 = 0$

D. $x^3 - 12x^2 + 44x - 49 = 0$

6 The motion of a particle undergoing simple harmonic motion is oscillatory, of which the amplitude of the motion remains constant.

However, in many real-world applications, oscillatory motions are often dampened out of necessity. The motion of a particle undergoing dampened harmonic motion is still oscillatory, but the amplitude of its motion reduces over time.

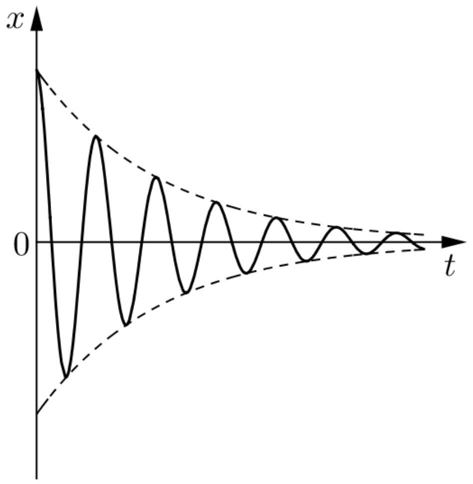
One such motion can be modelled by the equation:

$$x = e^{-\gamma t} \cos(\omega t)$$

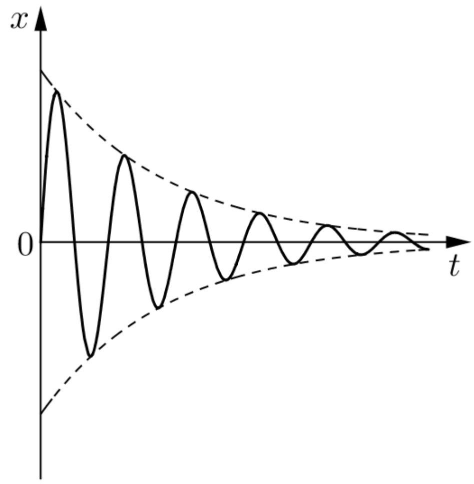
where γ and ω are positive constants, and x is the displacement of the particle from the centre of its oscillatory motion over time t.

Which of the following graphs best describes this model?

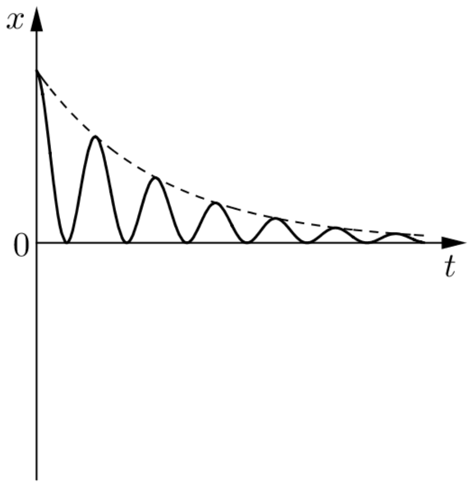
A.



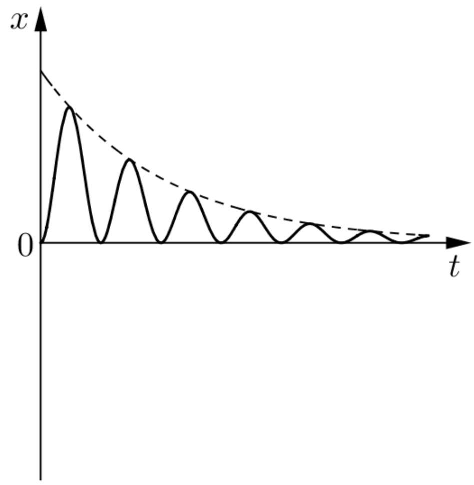
B.



C.

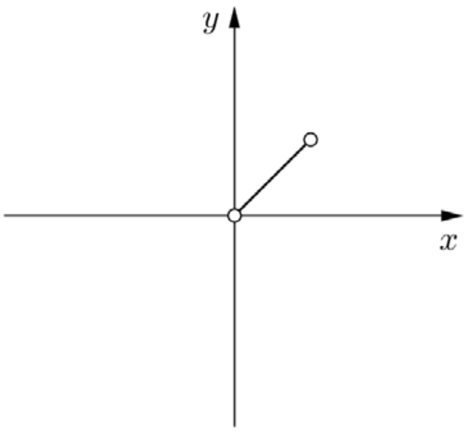


D.

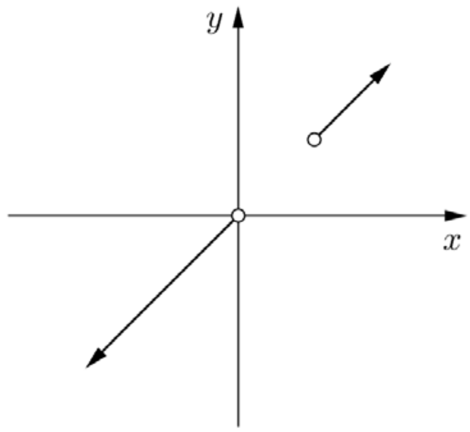


7 Which diagram best represents the solutions to the equation $\arg(z) = \arg(z + 2 + 2i)$?

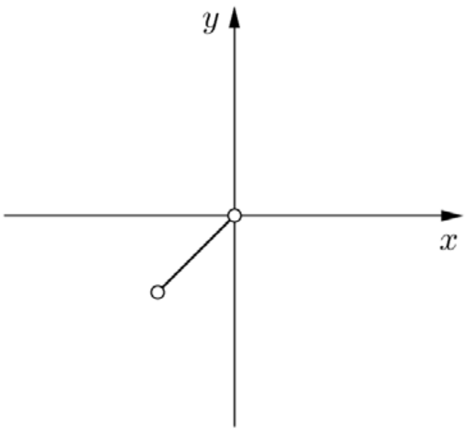
A.



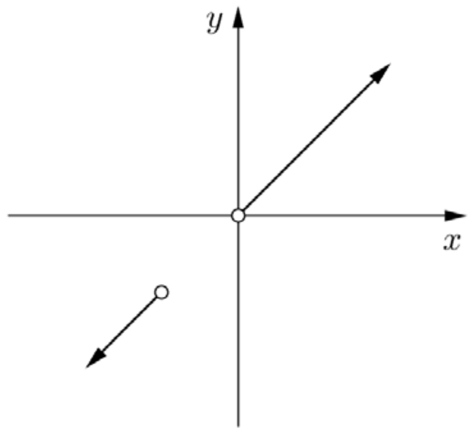
B.



C.



D.



---

8 It is given that $3+i$ is a root of $P(z) = z^3 + az^2 + bz + 10$ where *a* and *b* are real numbers. Which of the following expressions for $P(z)$ is correct?

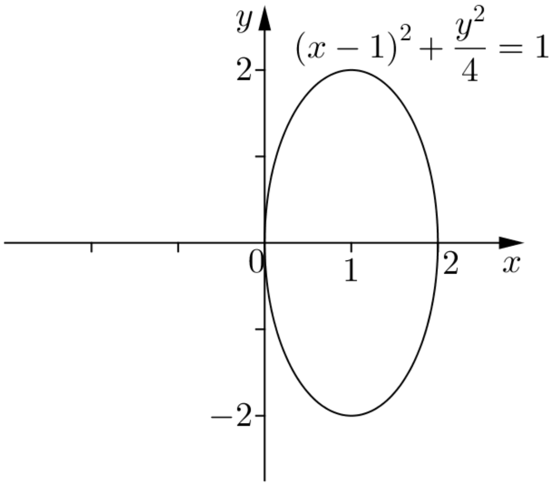
A. $P(z) = (z-1)\left(z^2 + 6z - 10\right)$

B. $P(z) = (z-1)\left(z^2 - 6z - 10\right)$

C. $P(z) = (z+1)\left(z^2 + 6z + 10\right)$

D. $P(z) = (z+1)\left(z^2 - 6z + 10\right)$

9 The region enclosed by the ellipse $(x-1)^2+\frac{y^2}{4}=1$ is rotated about the y-axis to form a solid.



Which integral represents the volume of the solid formed?

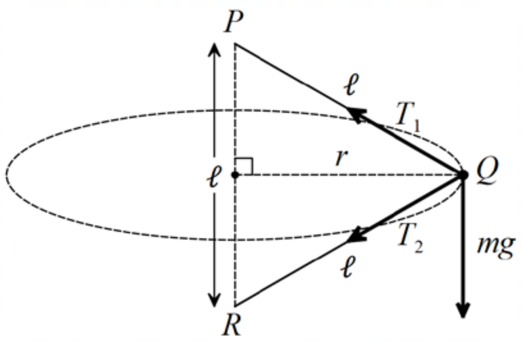
A. $\int_{-2}^{2} \pi\sqrt{4-y^2}\,dy$

B. $\int_{-2}^{2} \pi\sqrt{1-y^2}\,dy$

C. $\int_{-2}^{2} 2\pi\sqrt{4-y^2}\,dy$

D. $\int_{-2}^{2} 2\pi\sqrt{1-y^2}\,dy$

10 Two light inextensible strings *PQ* and *QR* each of length ℓ are attached to a particle of mass *m* at *Q*. The other ends *P* and *R* are fixed to two points in a vertical line such that *P* is at a distance ℓ above *R*. The particle moves in a horizontal circle with constant angular velocity ω such that both strings remain taut.



Taking g m/s^2 as the acceleration due to gravity, what are the tensions in the strings?

A. $T_1 = \frac{m}{2}\left(\ell\omega^2 + 2g\right)$ and $T_2 = \frac{m}{2}\left(\ell\omega^2 - 2g\right)$

B. $T_1 = \frac{m}{2}\left(\ell\omega^2 - 2g\right)$ and $T_2 = \frac{m}{2}\left(\ell\omega^2 + 2g\right)$

C. $T_1 = m\left(\ell\omega^2 + 2g\right)$ and $T_2 = m\left(\ell\omega^2 - 2g\right)$

D. $T_1 = m\left(\ell\omega^2 - 2g\right)$ and $T_2 = m\left(\ell\omega^2 + 2g\right)$

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate section of the writing booklet. Extra writing paper is available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 section of the writing booklet.

(a) Given $z = 3 + i$ and $w = 1 - 2i$, express $\frac{5z}{w}$ in $x + iy$ form, where x and y are real numbers. **2**

(b) (i) Write $\sqrt{3} + i$ in modulus-argument form. **2**

(ii) Find the smallest positive integer n such that $\left(\sqrt{3} + i\right)^n$ is a real number. **1**

(c) (i) Find real numbers a, b and c such that $\frac{4}{x^2(2-x)} \equiv \frac{ax+b}{x^2} + \frac{c}{2-x}$. **2**

(ii) Hence, or otherwise, find $\int \frac{4}{x^2(2-x)}\,dx$. **3**

(d) Find $\int \frac{dx}{\sqrt{1-4x-x^2}}$. **2**

(e) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{2+\cos x}$ using the substitution $t = \tan\frac{x}{2}$. **3**

End of Question 11

Question 12 (15 marks) Use the Question 12 section of the writing booklet.

(a) The equation $x^3 + x^2 + 2 = 0$ has roots α, β and γ.

(i) Show that $\alpha^2 + \beta^2 + \gamma^2 = 1$. **2**

(ii) Evaluate $\alpha^3 + \beta^3 + \gamma^3$. **2**

(iii) Evaluate $\alpha^4 + \beta^4 + \gamma^4$. **2**

(b) (i) Sketch the region on the Argand diagram where the following inequalities hold simultaneously. **3**

$$|z-1| \le \sqrt{2} \quad \text{and} \quad 0 \le \arg(z+i) \le \frac{\pi}{4}$$

(ii) Let w be the complex number of minimum modulus satisfying the inequalities in part (i). Express w in the form $x + iy$, where x and y are real number. **1**

(c) How many thirteen-letter arrangements can be made using the letters of the word BAULKHAMHILLS if:

(i) there are no restrictions? **1**

(ii) all seven of the repeated letters are grouped together in no particular order? **2**

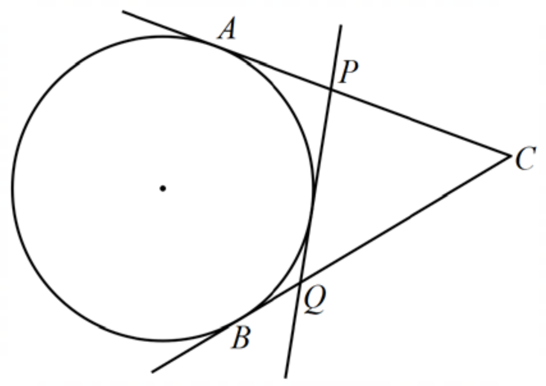
(iii) the vowels appear in alphabetical order but are separated by at least one consonant? **2**

End of Question 12

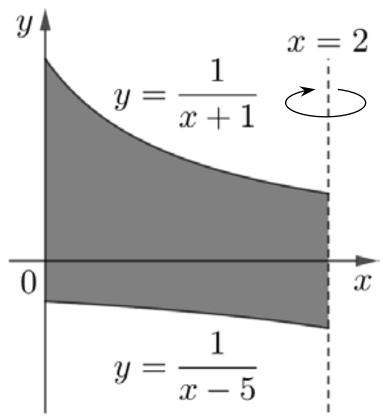
Question 13 (15 marks) Use the Question 13 section of the writing booklet.

(a) A and B are two points on a circle. Tangents at A and B meet at C. A third tangent on the minor arc AB cuts CA and CB at P and Q respectively, as shown in the diagram below. **2**

Copy the diagram into your writing booklet and show that the perimeter of ΔCPQ is independent of PQ.



(b) The region between the curves $y=\dfrac{1}{x+1}$, $y=\dfrac{1}{x-5}$, the y-axis and the line $x=2$ is rotated about the line $x=2$ to form a solid.



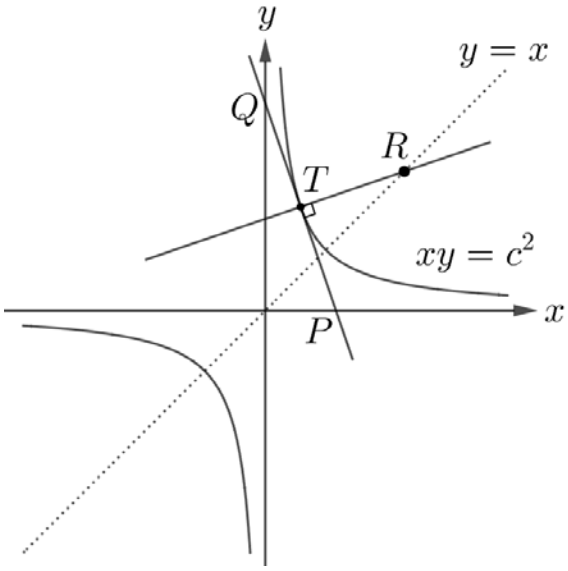
(i) Using the method of cylindrical shells, show that the volume of a typical shell of thickness Δx is given by $\Delta V = 12\pi\left(\dfrac{x-2}{x^2-4x-5}\right)\Delta x$. **2**

(ii) Hence, or otherwise, find the exact volume of the resulting solid. **3**

Question 13 continues on the next page

Question 13 (continued)

(c) The point $T\left(ct, \dfrac{c}{t}\right)$ lies on the hyperbola $xy = c^2$. The tangent at T meets the x-axis at P and the y-axis at Q. The normal at T meets the line $y = x$ at R.



You may assume that the tangent at T has equation $x + t^2 y = 2ct$ (Do NOT prove this.)

(i) Find the coordinates of P and Q. **2**

(ii) Find the equation of the normal at T. **2**

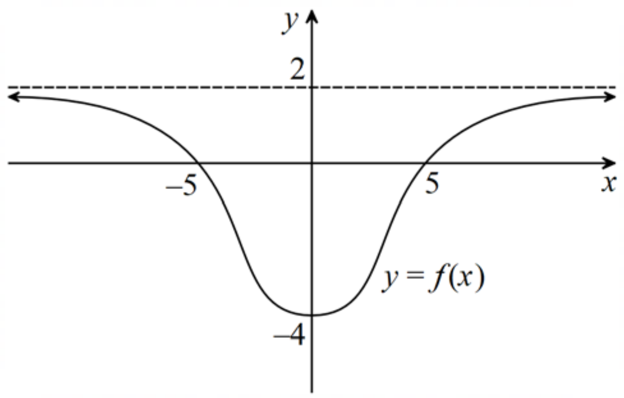
(iii) Show that the x-coordinate of R is $\dfrac{c}{t}\left(t^2 + 1\right)$. **2**

(iv) Prove that ΔPQR is isosceles. **2**

End of Question 13

Question 14 (15 marks) Use the Question 14 section of the writing booklet.

(a) The diagram shows the graph of $y = f(x)$.



Draw sketches of the graphs of the following on separate number planes:

(i) $y = \dfrac{1}{f(x)}$ **2**

(ii) $y = \left[f(x)\right]^2$ **2**

(iii) $y^2 = -f(x)$ **2**

(b) A particle of mass *m* kilograms is projected vertically upwards through a medium under the influence of gravity with an initial velocity of *U* m/s. In this medium, the particle experiences a resistive force of *mkv* newtons, where *k* is a positive constant and *v* is the particle's velocity in m/s. The acceleration due to gravity for this motion is g m/s^2.

It is known that the particle reaches its greatest height *H* metres in *T* seconds.

(i) Show that $T = \dfrac{1}{k}\ln\left(\dfrac{g+kU}{g}\right)$. **2**

(ii) Show that $\dfrac{dx}{dv} = -\dfrac{v}{g+kv}$, where *x* is the particle's displacement from its point of projection in metres. **1**

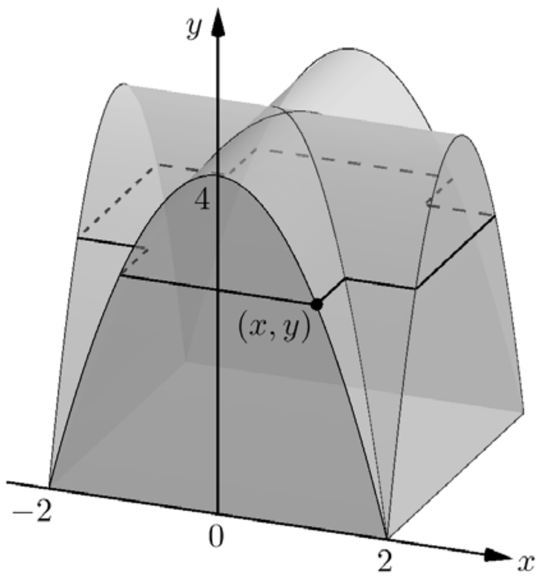
(iii) Hence, or otherwise, show that $U = kH + gT$. **2**

Question 14 continues on the next page

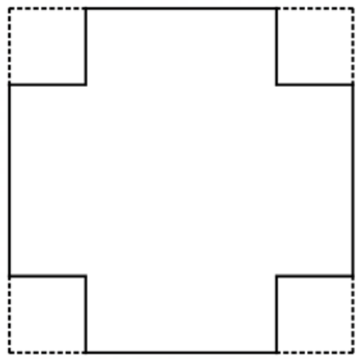
(c) A sandstone cap on the corner of a fence is show below. The solid is formed by intersecting two parabolic cylinders (a prism with uniform parabolic cross-sections).

On the front face, the equation of the parabola is $y = 4 - x^2$.

Slices parallel to the base of the solid are in the shape of a square with four smaller squares removed, one from each corner. The shape of a typical horizontal slice at height y is also shown below.



Sandstone cap



Shape of a typical slice

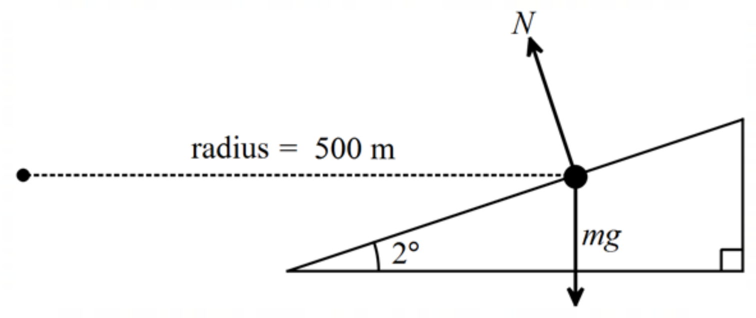
(i) Show that the area of a typical slice is $16 - 4 \times \left(2 - \sqrt{4 - y}\right)^2$ square units. **1**

(ii) Hence find the volume of the sandstone cap. **3**

End of Question 14

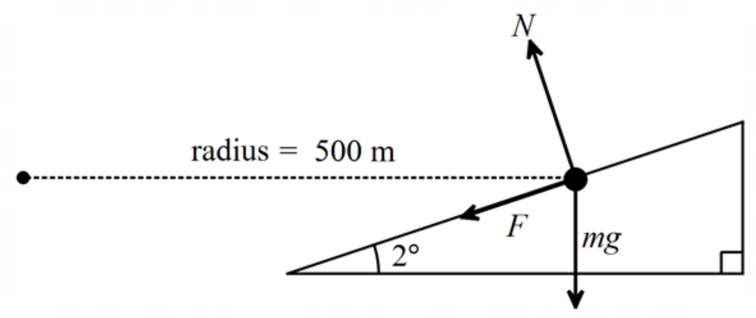
Question 15 (15 marks) Use the Question 15 section of the writing booklet.

(a) A 5000 kg truck is travelling around a circular portion of road of radius 500 metres. This portion of the road is banked at an angle of $2°$ to the horizontal. A cross-section of the road is shown in the diagram below. (Take g to be $10\ \text{m/s}^2$.)



(i) By resolving forces, determine the speed at which the truck must negotiate this circular portion of road such that the truck experiences no sideways friction. **2**

(ii) If the truck travels around this circular portion of road at a speed of 72 km/h, how much sideways friction (in newtons) is exerted on the tyres of the truck? **3**



(b) (i) Show that $\tan\left(A+\frac{\pi}{2}\right)=-\cot A$. **1**

(ii) Use mathematical induction to prove that $\tan\left[(2n+1)\frac{\pi}{4}\right]=(-1)^n$ for all integers $n \geq 1$. **3**

Question 15 continues on the next page

Question 15 (continued)

(c) Let $I_n = \int_0^1 \frac{x^n}{\sqrt{x+1}}\,dx$, where n is an integer and $n \geq 0$.

(i) Show that $I_0 = 2\sqrt{2} - 2$. **1**

(ii) Show that $I_{n-1} + I_n = \int_0^1 x^{n-1}\sqrt{x+1}\,dx$. **1**

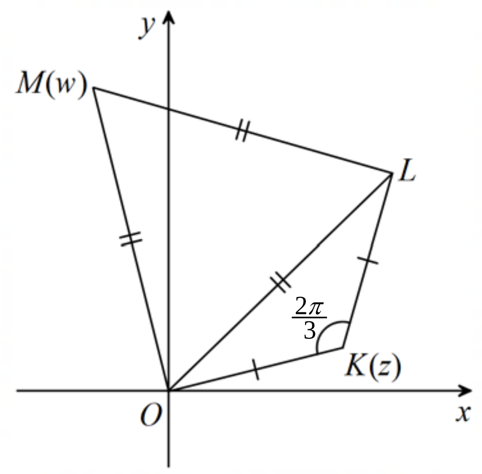
(iii) Use integration by parts to show that $I_n = \frac{2\sqrt{2} - 2nI_{n-1}}{2n+1}$ where n is an integer and $n \geq 1$. **2**

(iv) Hence, or otherwise, evaluate $\int_0^1 \frac{x^2}{\sqrt{x+1}}\,dx$. **2**

End of Question 15

Question 16 (15 marks) Use the Question 16 section of the writing booklet.

(a) On the Argand diagram below, the complex numbers z and w are represented by the points K and M respectively. It is given that ΔOKL is isosceles with $\angle OKL = \frac{2\pi}{3}$ as shown, and ΔOLM is equilateral. All angles are measured in radians.



(i) Show that $\angle MOK = \frac{\pi}{2}$. **1**

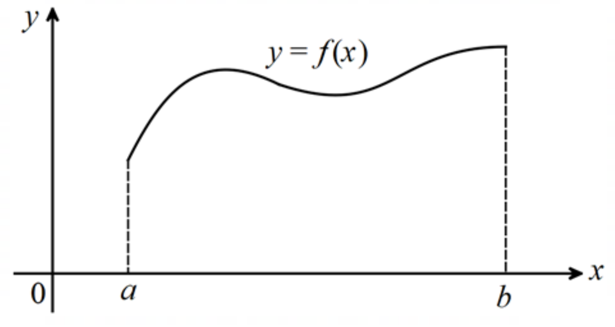
(ii) Show that $\left|\overrightarrow{OL}\right| = \sqrt{3} \times |z|$. **2**

(iii) Hence show that $3z^2 + w^2 = 0$. **2**

Question 16 continues on the next page

Question 16 (continued)

(b) The curve below represents $y = f(x)$, which is continuous and differentiable in the domain $a \le x \le b$.



The length of this curve, L, from $x = a$ to $x = b$ is given by $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$.

Now consider the curve $y = \frac{1}{2}\left(e^x + e^{-x}\right)$.

(i) Show that, for this curve, $1 + \left(\frac{dy}{dx}\right)^2 = \frac{\left(e^x + e^{-x}\right)^2}{4}$. **2**

(ii) Hence calculate the exact length of this curve from $x = 0$ to $x = 1$. **2**

(c) A polynomial of degree n is given by $P(x) = x^n + px - q$. It is given that the polynomial has a double root at $x = \alpha$.

(i) Show that $\alpha^{n-1} = -\frac{p}{n}$. **2**

(ii) Show that $\left(\frac{p}{n}\right)^n + \left(\frac{q}{n-1}\right)^{n-1} = 0$. **3**

(iii) Hence, or otherwise, deduce that the double root is $\frac{qn}{p(n-1)}$. **1**

End of Paper

YEAR 12 TRIAL EXAMINATION 2019
MATHEMATICS EXTENSION 2
MARKING GUIDELINES

Section I

Multiple-choice Answer Key

Question	Answer
1	B
2	B
3	A
4	C
5	D

Question	Answer
6	A
7	D
8	D
9	C
10	A

Questions 1 – 10

Sample solution	
1.	$i^{2019} = \left(i^4\right)^{504} \times i^3$ $= 1^{504} \times (-i)$ $= -i$
2.	The given graph is the portion of graph $xy = 1$ above the x-axis as well as its reflection in the x-axis, so replace y in the equation with $\lvert y \rvert$. The equation of the given graph is therefore $x\lvert y\rvert = 1$
3.	$4x^2 - 25y^2 = 100$ $\frac{x^2}{25} - \frac{y^2}{4} = 1$ $a^2 = 25 \Rightarrow a = 5$ $b^2 = 4 \Rightarrow b = 2$ Asymptotes are $y = \pm\frac{b}{a}x$, i.e. $y = \pm\frac{2}{5}x$
4.	$\frac{1+i}{1-i} = \frac{\sqrt{2}\left(\text{cis}\frac{\pi}{4}\right)}{\sqrt{2}\,\text{cis}\left(-\frac{\pi}{4}\right)}$ $= \text{cis}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right]$ $= \text{cis}\frac{\pi}{2}$ Multiplying by $\frac{1+i}{1-i}$ is therefore equivalent to multiplying by $\text{cis}\frac{\pi}{2}$, which causes an anticlockwise rotation by $\frac{\pi}{2}$ radians.
5.	For the new polynomial, $\sum\alpha = (2\alpha+\beta+\gamma)+(\alpha+2\beta+\gamma)+(\alpha+\beta+2\gamma)$ $= 4\alpha+4\beta+4\gamma$ $= 4(\alpha+\beta+\gamma)$ $= 4\times 3$ $= 12$ Only option D satisfies this property.
6.	$-1 \le \cos(\omega t) \le 1$ $-e^{-\gamma t} \le e^{-\gamma t}\cos(\omega t) \le e^{-\gamma t}$ When $t = 0$, $e^0 \cos 0 = 1$ Option A satisfies both of these properties.

Sample solution

7.

$\arg(z) = \arg(z+2+2i)$

$\arg(z) = \arg[z-(-2-2i)]$

Critical points are therefore $z=0$ and $z=-2-2i$.

Option D is the best representation of the solutions to the equation $\arg(z) = \arg(z+2+2i)$

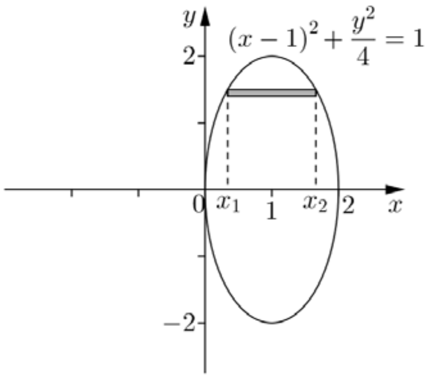
8.

Since a and b are real numbers, and $3+i$ is a root of $P(z)$, therefore $3-i$ must also be a root of $P(z)$.

Therefore, $[z-(3+i)][z-(3-i)]$ is a factor of $P(z)$.

$$\begin{aligned}[z-(3+i)][z-(3-i)] &= (z-3-i)(z-3+i)\\ &= (z-3)^2 - i^2\\ &= z^2-6z+9-(-1)\\ &= z^2-6z+10\end{aligned}$$

9.

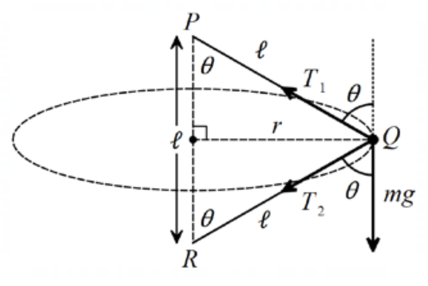


$$\begin{aligned}(x-1)^2+\frac{y^2}{4} &= 1\\ (x-1)^2 &= 1-\frac{y^2}{4}\\ x-1 &= \pm\sqrt{1-\frac{y^2}{4}}\\ x &= 1\pm\sqrt{1-\frac{y^2}{4}}\end{aligned}$$

$$\begin{aligned}\Delta V &= \pi\left(x_2^{\,2}-x_1^{\,2}\right)\Delta y\\ &= \pi\left[\left(1+\sqrt{1-\frac{y^2}{4}}\right)^2-\left(1-\sqrt{1-\frac{y^2}{4}}\right)^2\right]\Delta y\\ &= \pi\left(2\times 2\sqrt{1-\frac{y^2}{4}}\right)\Delta y\\ &= 4\pi\sqrt{1-\frac{y^2}{4}}\Delta y\end{aligned}$$

$$\begin{aligned}V &= \lim_{\Delta y\to 0}\sum_{y=-2}^{2} 4\pi\sqrt{1-\frac{y^2}{4}}\Delta y\\ &= \int_{-2}^{2} 4\pi\sqrt{1-\frac{y^2}{4}}\,dy\\ &= \int_{-2}^{2} 2\pi\sqrt{4-y^2}\,dy\end{aligned}$$

10.



ΔPQR is equilateral, $\therefore \cos\theta = \cos 60^\circ = \dfrac{1}{2}$

$\sin\theta = \dfrac{r}{\ell}$

Resolving vertically:

$$\begin{aligned}T_1\cos\theta - T_2\cos\theta - mg &= 0\\ T_1 - T_2 &= \frac{mg}{\cos\theta}\\ T_1 - T_2 &= 2mg\end{aligned}$$

Resolving horizontally:

$$\begin{aligned}T_1\sin\theta + T_2\sin\theta &= mr\omega^2\\ T_1 + T_2 &= \frac{mr\omega^2}{\sin\theta}\\ &= m\ell\omega^2\end{aligned}$$

Solving simultaneously,

$$T_1 = \frac{m\ell\omega^2+2mg}{2} = \frac{m}{2}\left(\ell\omega^2+2g\right) \qquad T_2 = \frac{m\ell\omega^2-2mg}{2} = \frac{m}{2}\left(\ell\omega^2-2g\right)$$

Section II

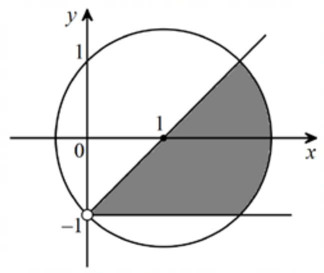
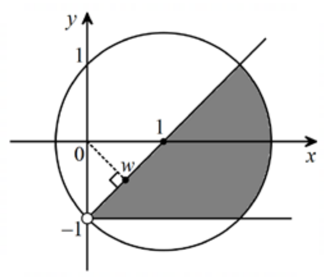
Question 11

Sample solution			Suggested marking criteria
(a)		$\frac{5z}{w}=\frac{5(3+i)}{1-2i}$ $=\frac{15+5i}{1-2i}$ $=\frac{(15+5i)(1+2i)}{(1-2i)(1+2i)}$ $=\frac{15+30i+5i+10i^2}{1^2-(2i)^2}$ $=\frac{5+35i}{5}$ $=1+7i$	• 2 – correct solution • 1 – realises the denominator
(b)	(i)	$\sqrt{3}+i=2\operatorname{cis}\frac{\pi}{6}$	• 2 – correct solution • 1 – correct modulus – correct argument (not necessarily principal argument)
	(ii)	$\left(\sqrt{3}+i\right)^n=\left(2\operatorname{cis}\frac{\pi}{6}\right)^n$ $=2^n\operatorname{cis}\frac{n\pi}{6}$ This is a real number if the argument is an integer multiple of π, the smallest positive integer n such that this happens is $n=6$.	• 1 – correct solution
(c)	(i)	$\frac{4}{x^2(2-x)}\equiv\frac{ax+b}{x^2}+\frac{c}{2-x}$ $4\equiv(ax+b)(2-x)+cx^2$ $4\equiv 2ax-ax^2+2b-bx+cx^2$ $4\equiv(c-a)x^2+(2a-b)x+2b$ Comparing coefficients: $2b=4$ $2a-b=0$ $c-a=0$ $b=2$ $2a-2=0$ $c-1=0$ $2a=2$ $c=1$ $a=1$	• 2 – correct solution • 1 – correctly finds two of the three coefficients
	(ii)	$\int\frac{4}{x^2(2-x)}dx=\int\frac{x+2}{x^2}+\frac{1}{2-x}dx$ $=\int\frac{1}{x}+2x^{-2}-\frac{-1}{2-x}dx$ $=\ln x-\frac{2}{x}-\ln(2-x)+c$ $=\ln\frac{x}{2-x}-\frac{2}{x}+c$	• 3 – correct solution • 2 – correctly integrates two of the three integrals listed below, or equivalent merit • 1 – correctly integrates one of the three integrals ($\frac{1}{x}$, $2x^{-2}$ or $\frac{1}{2-x}$), or equivalent merit

Question 11 (continued)

<table>
<tr><th colspan="2">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td>(d)</td><td>$$\begin{aligned}\int \frac{dx}{\sqrt{1-4x-x^2}} &= \int \frac{dx}{\sqrt{-\left(x^2+4x-1\right)}} \\ &= \int \frac{dx}{\sqrt{-\left[\left(x+2\right)^2-5\right]}} \\ &= \int \frac{dx}{\sqrt{5-\left(x+2\right)^2}} \\ &= \sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+c\end{aligned}$$</td><td>• 2 – correct solution
• 1 – attempts to complete the square and reduces the integral to a “standard” integral</td></tr>
<tr><td>(e)</td><td>$$\begin{aligned}t &= \tan\frac{x}{2} \\ \frac{dt}{dx} &= \frac{1}{2}\sec^2\frac{x}{2} \\ &= \frac{1}{2}\left(1+\tan^2\frac{x}{2}\right) \\ &= \frac{1+t^2}{2} \\ \frac{dx}{dt} &= \frac{2}{1+t^2}\end{aligned}$$
When $x=0, t=0$.
When $x=\frac{\pi}{2}, t=1$.
$$\begin{aligned}\int_0^{\frac{\pi}{2}} \frac{dx}{2+\cos x} &= \int_0^1 \frac{\frac{2dt}{1+t^2}}{2+\frac{1-t^2}{1+t^2}} \\ &= \int_0^1 \frac{2dt}{2\left(1+t^2\right)+\left(1-t^2\right)} \\ &= \int_0^1 \frac{2dt}{3+t^2} \\ &= 2\times\left[\frac{1}{\sqrt{3}}\tan^{-1}\frac{t}{\sqrt{3}}\right]_0^1 \\ &= 2\times\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{3}}\tan^{-1}0\right) \\ &= \frac{2}{\sqrt{3}}\times\frac{\pi}{6} \\ &= \frac{\pi}{3\sqrt{3}}\end{aligned}$$</td><td>• 3 – correct solution
• 2 – applies the substitution to express the integral in terms of t
• 1 – applies the substitution to partially change the integral to an integral in terms of t</td></tr>
</table>

Question 12

<table>
<tr><th colspan="3">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td>(a)</td><td>(i)</td><td>$\sum\alpha=-1$
$\sum\alpha\beta=0$
$\sum\alpha^2=\left(\sum\alpha\right)^2-2\sum\alpha\beta$
$=(-1)^2-2\times0$
$=1$</td><td>• 2 – correct solution
• 1 – finds $\sum\alpha$</td></tr>
<tr><td></td><td>(ii)</td><td>α,β and γ are roots of $x^3+x^2+2=0$
$\alpha^3+\alpha^2+2=0$
$\beta^3+\beta^2+2=0$
$\gamma^3+\gamma^2+2=0 \quad +$
$\left(\alpha^3+\beta^3+\gamma^3\right)+\left(\alpha^2+\beta^2+\gamma^2\right)+6=0$
$\left(\alpha^3+\beta^3+\gamma^3\right)+1+6=0$
$\alpha^3+\beta^3+\gamma^3=-7$</td><td>• 2 – correct solution
• 1 – forms an expression involving $\sum\alpha^3$ and $\sum\alpha^2$</td></tr>
<tr><td></td><td>(iii)</td><td>$\alpha^4+\alpha^3+2\alpha=0$
$\beta^4+\beta^3+2\alpha=0$
$\gamma^4+\gamma^3+2\alpha=0 \quad +$
$\left(\alpha^4+\beta^4+\gamma^4\right)+\left(\alpha^3+\beta^3+\gamma^3\right)+2\left(\alpha+\beta+\gamma\right)=0$
$\left(\alpha^4+\beta^4+\gamma^4\right)+(-7)+2\times(-1)=0$
$\alpha^4+\beta^4+\gamma^4=9$</td><td>• 2 – correct solution
• 1 – forms an expression involving $\sum\alpha^3$, $\sum\alpha^2$ and $\sum\alpha$</td></tr>
<tr><td>(b)</td><td>(i)</td><td>
</td><td>• 3 – correct region
• 2 – correct boundary
• 1 – correctly sketches one of the two regions of the given inequalities</td></tr>
<tr><td></td><td>(ii)</td><td>For w to lie in the region in part (i) and be of minimum modulus, w must be in the position as shown below:


By inspection, $w=\frac{1}{2}-\frac{1}{2}i$.</td><td>• 1 – correct answer</td></tr>
</table>

Question 12 (continued)

<table>
<tr><th colspan="3">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td rowspan="3">(c)</td><td>(i)</td><td>$\dfrac{13!}{2!2!3!} = 259459200$ ways</td><td>• 1 – correct answer</td></tr>
<tr><td>(ii)</td><td>There are $\dfrac{7!}{2!2!3!}$ ways of arranging the seven repeated letter, then treating this as a group, there are $7!$ ways of arranging this group of repeated letter and the 6 remaining letters.

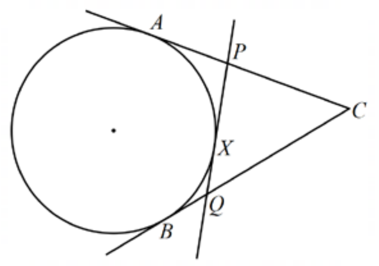
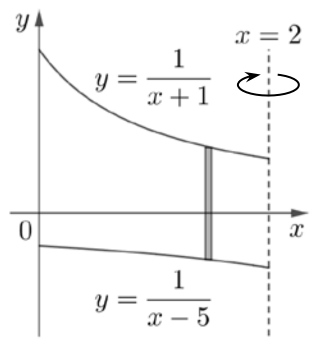
Therefore, there are $\dfrac{7!}{2!2!3!} \times 7! = 1058400$ different arrangements.</td><td>• 2 – correct solution
• 1 – correctly arranges the repeated letters</td></tr>
<tr><td>(iii)</td><td>Let v be a vowel and c be a consonant of the word BAULKHAMHILLS, start with a string of the nine consonants, like so:

_ c _ c _ c _ c _ c _ c _ c _ c _ c _

With this string of consonants, there are ten spaces, choose any four of them to insert the vowels A, A, I and U in that order.

Taking into account the number of possible arrangements of the consonants, this can be done in $\dfrac{9!}{2!3!} \times {}^{10}C_4 = 6350400$ ways.</td><td>• 2 – correct solution
• 1 – considers some of the valid cases
– progress towards solution using valid reasoning</td></tr>
</table>

Question 13

Sample solution		Suggested marking criteria
(a)	 Let X be the point of contact of the third tangent. $CA = CB$ (tangents from an external point are equal in length) $PA = PX$ (tangents from an external point are equal in length) $QB = QX$ (tangents from an external point are equal in length) The perimeter of $\Delta CPQ = CP + PQ + CQ$ $= CP + (PX + QX) + CQ$ $= (CP + PA) + (QB + CQ)$ (since $PX = PA$ and $QX = QB$) $= CA + CB$, which is independent of PQ.	• 2 – correct proof • 1 – one correct circle geometry reasoning towards the correct proof
(b)	(i)  $\Delta V = 2\pi(2-x)\left(\frac{1}{x+1} - \frac{1}{x-5}\right)\Delta x$ $= 2\pi(2-x)\left[\frac{x-5-(x+1)}{(x+1)(x-5)}\right]\Delta x$ $= 2\pi(2-x)\left(\frac{-6}{x^2-4x-5}\right)\Delta x$ $= 12\pi\left(\frac{x-2}{x^2-4x-5}\right)\Delta x$	• 2 – correct solution • 1 – finds a correct expression for ΔV
	(ii) $\Delta V = 12\pi\left(\frac{x-2}{x^2-4x-5}\right)\Delta x$ $V = \lim_{\Delta x \to 0} \sum_{x=0}^{2} 12\pi\left(\frac{x-2}{x^2-4x-5}\right)\Delta x$ $= \int_0^2 12\pi\left(\frac{x-2}{x^2-4x-5}\right)dx$ $= 6\pi\int_0^2 \left(\frac{2x-4}{x^2-4x-5}\right)dx$ $= 6\pi\left[\ln\lvert x^2-4x-5\rvert\right]_0^2$ $= 6\pi\left(\ln\lvert -9\rvert - \ln\lvert -5\rvert\right)$ $= 6\pi\ln\frac{9}{5}$ cubic units	• 3 – correct solution • 2 – correct primitive • 1 – establishes correct sum of infinitesimals
(c)	(i) The tangent at T has equation $x + t^2 y = 2ct$. When $y = 0, x = 2ct$, $\therefore P = (2ct, 0)$ When $x = 0, y = \frac{2c}{t}$, $\therefore Q = \left(0, \frac{2c}{t}\right)$	• 2 – correct solution • 1 – correctly finds P or Q

Question 13 (continued)

<table>
<tr><th colspan="3">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td rowspan="3">(c)</td><td>(ii)

$$xy = c^2$$
$$y = \frac{c^2}{x}$$
$$\frac{dy}{dx} = \frac{-c^2}{x^2}$$

At T,

$$\frac{dy}{dx} = -\frac{c^2}{(ct)^2}$$
$$= -\frac{1}{t^2}$$</td><td>Equation of normal at T:

$$y - \frac{c}{t} = t^2(x - ct)$$
$$y - \frac{c}{t} = t^2 x - ct^3$$
$$ty - c = t^3 x - ct^4$$
$$ct^4 - c = t^3 x - ty$$
$$t^3 x - ty = c(t^4 - 1)$$</td><td>

- 2 – correct solution
- 1 – correct gradient for the normal</td></tr>
<tr><td colspan="2">(iii)

$$t^3 x - ty = c(t^4 - 1)$$
$$t^3 x - tx = c(t^2 + 1)(t^2 - 1)$$
$$xt(t^2 - 1) = c(t^2 + 1)(t^2 - 1)$$
$$xt = c(t^2 + 1)$$
$$x = \frac{c}{t}(t^2 + 1)$$</td><td>

- 2 – correct solution
- 1 – attempts to use simultaneous equations to find the x-coordinate of R</td></tr>
<tr><td colspan="2">(iv)

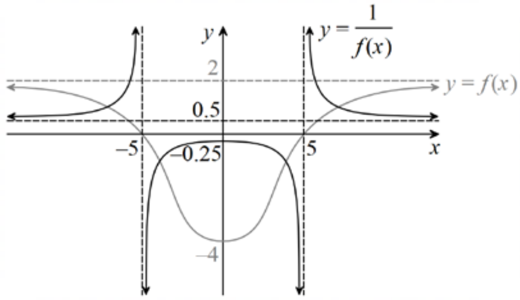
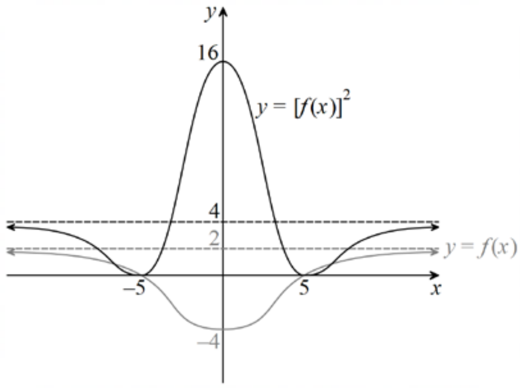
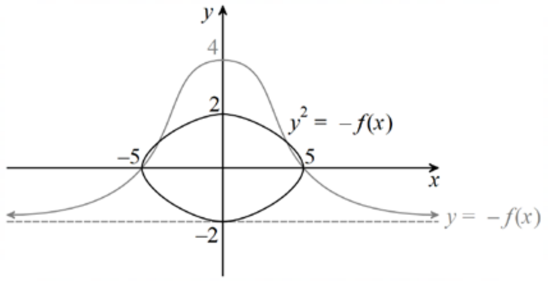
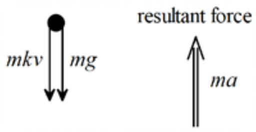
$$d_{PR} = \sqrt{\left[\frac{c}{t}(t^2 + 1) - 2ct\right]^2 + \left[\frac{c}{t}(t^2 + 1)\right]^2}$$
$$= \sqrt{\left(ct + \frac{c}{t} - 2ct\right)^2 + \left(ct + \frac{c}{t}\right)^2}$$
$$= \sqrt{\left(-ct + \frac{c}{t}\right)^2 + \left(ct + \frac{c}{t}\right)^2}$$
$$= \sqrt{c^2t^2 - 2c^2 + \frac{c^2}{t^2} + c^2t^2 + 2c^2 + \frac{c^2}{t^2}}$$
$$= \sqrt{2c^2t^2 + \frac{2c^2}{t^2}}$$

$$d_{QR} = \sqrt{\left[\frac{c}{t}(t^2 + 1)\right]^2 + \left[\frac{c}{t}(t^2 + 1) - \frac{2c}{t}\right]^2}$$
$$= \sqrt{\left(ct + \frac{c}{t}\right)^2 + \left(ct + \frac{c}{t} - \frac{2c}{t}\right)^2}$$
$$= \sqrt{\left(ct + \frac{c}{t}\right)^2 + \left(ct - \frac{c}{t}\right)^2}$$
$$= \sqrt{c^2t^2 + 2c^2 + \frac{c^2}{t^2} + c^2t^2 - 2c^2 + \frac{c^2}{t^2}}$$
$$= \sqrt{2c^2t^2 + \frac{2c^2}{t^2}}$$

Since $d_{PR} = d_{QR}$, ΔPQR is isosceles.</td><td>

- 2 – correct solution
- 1 – finds d_{PR} or d_{QR}
  - – establishes T as the midpoint of PQ
  - – calculates the gradient of PQ or QR</td></tr>
</table>

Question 14

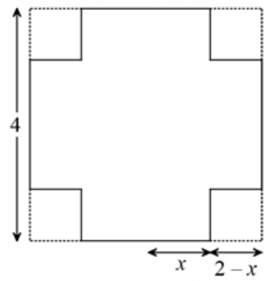
<table>
<tr><th colspan="2">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td rowspan="3">(a)</td><td>(i)

</td><td>• 2 – correct solution
• 1 – significant progress towards solution</td></tr>
<tr><td>(ii)

</td><td>• 2 – correct solution
• 1 – correctly identifies the turning points</td></tr>
<tr><td>(iii)

</td><td>• 2 – correct solution
• 1 – sketches $y^2 = f(x)$
– sketches $y = \sqrt{-f(x)}$
– sketches $y = -\sqrt{-f(x)}$
– correctly positions the graph of $y^2 = f(x)$ but missing important features, or equivalent merit</td></tr>
<tr><td>(b)</td><td>(i)


$ma = -mg - mkv$
$a = -(g + kv)$
$\frac{dv}{dt} = -(g + kv)$
$\frac{dt}{dv} = -\frac{1}{g + kv}$
$\int_0^T dt = \int_U^0 -\frac{1}{g + kv}\,dv$
$T = \frac{1}{k}\int_0^U \frac{k}{g + kv}\,dv$
$= \frac{1}{k}\left[\ln(g + kv)\right]_0^U$
$= \frac{1}{k}\left[\ln(g + kU) - \ln g\right]$
$= \frac{1}{k}\ln\left(\frac{g + kU}{g}\right)$</td><td>• 2 – correct solution
• 1 – correctly integrates $-\frac{1}{g + kv}$</td></tr>
</table>

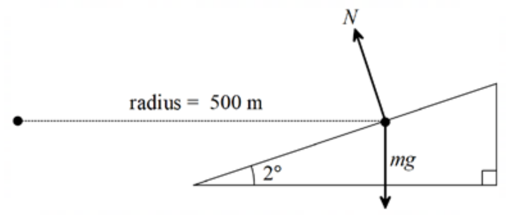
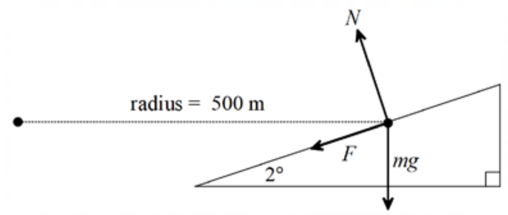
Question 14 (continued)

<table>
<tr><th colspan="2">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td rowspan="2">(b)</td><td>(ii)
$$a = -(g + kv)$$
$$v\frac{dv}{dx} = -(g + kv)$$
$$\frac{dv}{dx} = -\frac{g + kv}{v}$$
$$\frac{dx}{dv} = -\frac{v}{g + kv}$$</td><td>• 1 – correct solution</td></tr>
<tr><td>(iii)
$$\frac{dx}{dv} = -\frac{v}{g + kv}$$
$$\int_0^H dx = \int_U^0 -\frac{v}{g + kv}\,dv$$
$$H = \frac{1}{k}\int_0^U \frac{kv}{g + kv}\,dv$$
$$= \frac{1}{k}\int_0^U \frac{g + kv - g}{g + kv}\,dv$$
$$= \frac{1}{k}\int_0^U 1 - \frac{g}{g + kv}\,dv$$
$$= \frac{1}{k}\int_0^U 1 - \frac{g}{k} \times \frac{k}{g + kv}\,dv$$
$$= \frac{1}{k}\left[v - \frac{g}{k}\ln(g + kv)\right]_0^U$$
$$= \frac{1}{k}\left\{\left[U - \frac{g}{k}\ln(g + kU)\right] - \left[0 - \frac{g}{k}\ln g\right]\right\}$$
$$= \frac{1}{k}\left[U - \frac{g}{k}\ln\left(\frac{g + kU}{g}\right)\right]$$
$$= \frac{1}{k}(U - gT)$$
$$kH = U - gT$$
$$U = kH + gT$$</td><td>• 2 – correct solution
• 1 – correctly integrates $-\frac{v}{g + kv}$</td></tr>
</table>

Question 14 (continued)

Sample solution			Suggested marking criteria
(c)	(i)	 $y = 4 - x^2$ $x^2 = 4 - y$ $x = \sqrt{4-y}$ Area of slice $= 4^2 - 4\times(2-x)^2$ $= 16 - 4\times\left(2-\sqrt{4-y}\right)^2$	• 1 – correct solution
	(ii)	$\Delta V = 4^2 - 4\times\left(2-\sqrt{4-y}\right)^2 \Delta y$ $V = \lim\limits_{\Delta y \to 0} \sum\limits_{y=0}^{4} \left[16 - 4\times\left(2-\sqrt{4-y}\right)^2\right]\Delta y$ $= \int_0^4 \left[16 - 4\times\left(2-\sqrt{4-y}\right)^2\right] dy$ $= \int_0^4 \left[16 - 4\times\left(4 - 4\sqrt{4-y} + 4 - y\right)\right] dy$ $= \int_0^4 \left[16 - 4\times\left(8 - 4\sqrt{4-y} - y\right)\right] dy$ $= \int_0^4 \left(16 - 32 + 16\sqrt{4-y} + 4y\right) dy$ $= \int_0^4 \left(-16 + 16\sqrt{4-y} + 4y\right) dy$ $= \int_0^4 \left(-16 + 16(4-y)^{\frac{1}{2}} + 4y\right) dy$ $= \left[-16y + \dfrac{16(4-y)^{\frac{3}{2}}}{-\left(\frac{3}{2}\right)} + 2y^2\right]_0^4$ $= \left[-16y - \dfrac{32(4-y)^{\frac{3}{2}}}{3} + 2y^2\right]_0^4$ $= (-64+32) + \left(\dfrac{32\times 4^{\frac{3}{2}}}{3}\right)$ $= 53\frac{1}{3}$ cubic units	• 3 – correct solution • 2 – correct primitive • 1 – correct integral

Question 15

		Sample solution	Suggested marking criteria
(a)	(i)	 Resolving forces vertically: $N\cos 2° - mg = 0$ $N\cos 2° = 50000$ Resolving forces horizontally: $N\sin 2° = \dfrac{mv^2}{r}$ $N\sin 2° = \dfrac{5000v^2}{500}$ $= 10v^2$ $\dfrac{N\sin 2°}{N\cos 2°} = \dfrac{10v^2}{50000}$ $\tan 2° = \dfrac{v^2}{5000}$ $v = \sqrt{5000\tan 2°}$ $= 13.21\text{ m/s} \quad (2\text{ d.p.})$	• 2 – correct solution • 1 – resolves the forces vertically and horizontally, or equivalent merit
	(ii)	 Resolving forces vertically: $N\cos 2° - F\sin 2° - mg = 0$ $N\cos 2° - F\sin 2° = 50000$ Resolving forces horizontally: $N\sin 2° + F\cos 2° = \dfrac{mv^2}{r}$ $N\sin 2° + F\cos 2° = \dfrac{5000\times 20^2}{500}$ Eliminating $N\sin 2°\cos 2°$: $N\sin 2°\cos 2° - F\sin^2 2° = 50000\sin 2°$ $N\sin 2°\cos 2° + F\cos^2 2° = 4000\cos 2° \qquad -$ $F\cos^2 2° + F\sin^2 2° = 4000\cos 2° - 50000\sin 2°$ $F = 2252.6\text{ N} \quad (1\text{ d.p.})$	• 3 – correct solution • 2 – resolves forces vertically and horizontally, or equivalent merit • 1 – resolves forces in one direction
(b)	(i)	$\tan\left(A + \dfrac{\pi}{2}\right) = \cot\left[\dfrac{\pi}{2} - \left(A + \dfrac{\pi}{2}\right)\right]$ $= \cot(-A)$ $= \dfrac{1}{\tan(-A)}$ $= \dfrac{1}{-\tan A}$ ($\tan x$ is an odd function) $= -\cot A$	• 1 – correct solution

Question 15 (continued)

Sample solution			Suggested marking criteria
(b)	(ii)	Let $S(n)$ be the statement that $\tan\left[(2n+1)\frac{\pi}{4}\right]=(-1)^n$. Show $S(1)$ is true: $\text{LHS}=\tan\left(\frac{3\pi}{4}\right)=-\tan\frac{\pi}{4}=-1$ $\text{RHS}=(-1)^1=-1$ Since LHS = RHS, $\therefore S(1)$ is true. Assume $S(k)$ is true: i.e. $\tan\left[(2k+1)\frac{\pi}{4}\right]=(-1)^k$ Prove that $S(k+1)$ is true: i.e. $\tan\left[(2k+3)\frac{\pi}{4}\right]=(-1)^{k+1}$ $\text{LHS}=\tan\left[(2k+3)\frac{\pi}{4}\right]$ $=\tan\left[(2k+1)\frac{\pi}{4}+2\times\frac{\pi}{4}\right]$ $=\tan\left[(2k+1)\frac{\pi}{4}+\frac{\pi}{2}\right]$ $=-\cot\left[(2k+1)\frac{\pi}{4}\right]$ $=-\frac{1}{\tan\left[(2k+1)\frac{\pi}{4}\right]}$ $=-\frac{1}{(-1)^k}$ $=\frac{-1}{(-1)^k}\times\frac{(-1)^k}{(-1)^k}$ $=\frac{(-1)^{k+1}}{(-1)^{2k}}$ $=\frac{(-1)^{k+1}}{\left[(-1)^2\right]^k}$ $=\frac{(-1)^{k+1}}{1^k}$ $=(-1)^{k+1}$ = RHS $\therefore S(k+1)$ is true if $S(k)$ is assumed true. Since $S(1)$ is proven true, then by the principle of mathematical induction, $S(n)$ is true for all integers $n\geq 1$.	• 3 – correct solution • 2 – uses the inductive hypothesis to attempt to prove the result inductively • 1 – showing the result to be true for $n=1$

Question 15 (continued)

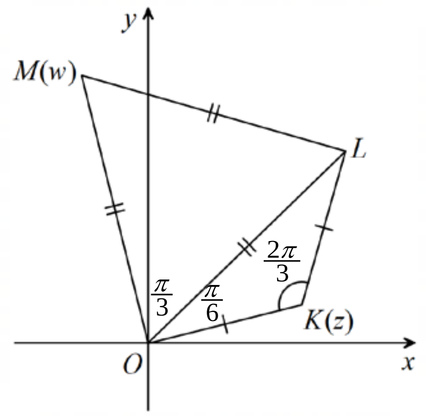
Sample solution		Suggested marking criteria
(c)	(i) $$I_0 = \int_0^1 \frac{1}{\sqrt{x+1}}\,dx$$ $$= \int_0^1 (x+1)^{-\frac{1}{2}}\,dx$$ $$= \left[2(x+1)^{\frac{1}{2}}\right]_0^1$$ $$= 2 \times 2^{\frac{1}{2}} - 2$$ $$= 2\sqrt{2} - 2$$	• 1 – correct solution
	(ii) $$I_{n-1} + I_n = \int_0^1 \frac{x^{n-1}}{\sqrt{x+1}}\,dx + \int_0^1 \frac{x^n}{\sqrt{x+1}}\,dx$$ $$= \int_0^1 \frac{x^{n-1}}{\sqrt{x+1}} + \frac{x^n}{\sqrt{x+1}}\,dx$$ $$= \int_0^1 \frac{x^{n-1} + x^n}{\sqrt{x+1}}\,dx$$ $$= \int_0^1 \frac{x^{n-1}(1+x)}{\sqrt{x+1}}\,dx$$ $$= \int_0^1 x^{n-1}\sqrt{x+1}\,dx$$	• 1 – correct solution
	(iii) Using integration by parts, $$I_{n-1} + I_n = \left[\frac{x^n\sqrt{x+1}}{n}\right]_0^1 - \int_0^1 \frac{x^n}{2n\sqrt{x+1}}\,dx \qquad u = \sqrt{x+1} \qquad du = \frac{dx}{2\sqrt{x+1}}$$ $$= \frac{\sqrt{2}}{n} - \frac{1}{2n}\int_0^1 \frac{x^n}{\sqrt{x+1}}\,dx \qquad v = \frac{x^n}{n} \qquad dv = x^{n-1}dx$$ $$= \frac{\sqrt{2}}{n} - \frac{1}{2n} I_n$$ $$I_n + \frac{1}{2n} I_n = \frac{\sqrt{2}}{n} - I_{n-1}$$ $$I_n\left(1 + \frac{1}{2n}\right) = \frac{\sqrt{2}}{n} - I_{n-1}$$ $$I_n(2n+1) = 2\sqrt{2} - 2nI_{n-1}$$ $$I_n = \frac{2\sqrt{2} - 2nI_{n-1}}{2n+1}$$	• 2 – correct solution • 1 – attempts to use integration by parts towards the required result

Question 15 (continued)

Sample solution			Suggested marking criteria
(c)	(iv)	$$\int_0^1 \frac{x^2}{\sqrt{x+1}}\,dx = I_2$$ $$= \frac{2\sqrt{2} - 4I_1}{5}$$ $$= \frac{2\sqrt{2} - 4\times\left(\frac{2\sqrt{2} - 2I_0}{3}\right)}{5}$$ $$= \frac{2\sqrt{2} - 4\times\left[\frac{2\sqrt{2} - 2\times\left(2\sqrt{2} - 2\right)}{3}\right]}{5}$$ $$= \frac{2\sqrt{2} - 4\times\left[\frac{2\sqrt{2} - 4\sqrt{2} + 4}{3}\right]}{5}$$ $$= \frac{2\sqrt{2} - 4\times\left[\frac{4 - 2\sqrt{2}}{3}\right]}{5}$$ $$= \frac{6\sqrt{2} - 4\times\left(4 - 2\sqrt{2}\right)}{15}$$ $$= \frac{6\sqrt{2} - 16 + 8\sqrt{2}}{15}$$ $$= \frac{14\sqrt{2} - 16}{15}$$	• 2 – correct solution • 1 – one correct application of the reduction formula – finds I_1

Question 16

<table>
<tr><th colspan="3">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td>(a)</td><td>(i)</td><td>



In ΔOKL,

$\angle KOL = \angle KLO$ (angles opposite equal sides of a triangle are equal)

$\angle KOL + \angle KLO + \angle OKL = \pi$ (angle sum of ΔOKL is π radians)

$$\angle KOL + \angle KOL + \frac{2\pi}{3} = \pi$$

$$2\angle KOL = \frac{\pi}{3}$$

$$\angle KOL = \frac{\pi}{6}$$

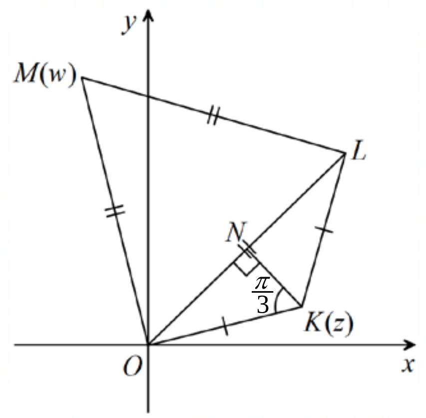
In ΔOML, $\angle MOL = \frac{\pi}{3}$ (angles in an equilateral triangle)

$$\begin{aligned}\angle MOK &= \angle MOL + \angle KOL \\ &= \frac{\pi}{3} + \frac{\pi}{6} \\ &= \frac{\pi}{2}\end{aligned}$$

</td><td>

• 1 – correct solution

</td></tr>
<tr><td></td><td>(ii)</td><td>



Since ΔOKL is isosceles, by symmetry, if KN is an altitude of the triangle,

N bisect OL and $\angle OKN = \frac{\pi}{3}$.

$$\sin\frac{\pi}{3} = \frac{|\overrightarrow{ON}|}{|\overrightarrow{OK}|}$$

$$\frac{\sqrt{3}}{2} = \frac{\frac{1}{2}|\overrightarrow{OL}|}{|z|}$$

$$\sqrt{3} = \frac{|\overrightarrow{OL}|}{|z|}$$

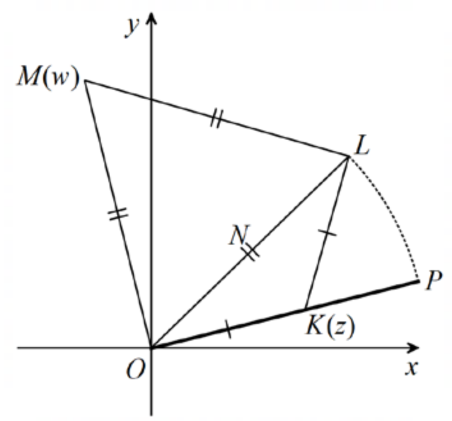
$$|\overrightarrow{OL}| = \sqrt{3} \times |z|$$

</td><td>

• 2 – correct solution
• 1 – uses trigonometry to attempt to show the required result

</td></tr>
</table>

Question 16 (continued)

Sample solution		**Suggested marking criteria**
(a)	(iii)  Suppose $\overrightarrow{OP}$ is the vector in the direction of $\overrightarrow{OK}$ with length equal to $\overrightarrow{OL}$. Since $\left\lvert\overrightarrow{OL}\right\rvert = \sqrt{3}\times\lvert z\rvert$, therefore $\left\lvert\overrightarrow{OP}\right\rvert = \sqrt{3}\times\lvert z\rvert$, i.e. $\overrightarrow{OP} = \sqrt{3}\times\overrightarrow{OK}$. $\overrightarrow{OM} = i\overrightarrow{OP}$ $\overrightarrow{OM} = i\sqrt{3}\times\overrightarrow{OK}$ $w = i\sqrt{3}\times z$ $w^2 = -3z^2$ $3z^2 + w^2 = 0$	• 2 – correct solution • 1 – expresses w in terms of z
(b)	(i) $y = \frac{1}{2}\left(e^x + e^{-x}\right)$ $\frac{dy}{dx} = \frac{1}{2}\left(e^x - e^{-x}\right)$ $1+\left(\frac{dy}{dx}\right)^2 = 1+\left[\frac{1}{4}\left(e^x - e^{-x}\right)^2\right]$ $= 1+\frac{\left(e^x\right)^2 - 2 + \left(e^{-x}\right)^2}{4}$ $= \frac{4+\left(e^x\right)^2 - 2 + \left(e^{-x}\right)^2}{4}$ $= \frac{\left(e^x\right)^2 + 2 + \left(e^{-x}\right)^2}{4}$ $= \frac{\left(e^x + e^{-x}\right)^2}{4}$	• 2 – correct solution • 1 – finds $\frac{dy}{dx}$
	(ii) $L = \int_0^1 \sqrt{\frac{\left(e^x + e^{-x}\right)^2}{4}}dx$ $= \int_0^1 \frac{e^x + e^{-x}}{2}dx$ $= \frac{1}{2}\left[e^x - e^{-x}\right]_0^1$ $= \frac{1}{2}\left[\left(e - e^{-1}\right) - \left(e^0 - e^0\right)\right]$ $= \frac{1}{2}\left(e - \frac{1}{e}\right)$ units	• 2 – correct solution • 1 – correct integration

Question 16 (continued)

Sample solution			**Suggested marking criteria**
(c)	(i)	$P(x)=x^n+px-q$ $P'(x)=nx^{n-1}+p$ $P'(\alpha)=0$ $n\alpha^{n-1}+p=0$ $n\alpha^{n-1}=-p$ $\alpha^{n-1}=-\frac{p}{n}$	• 2 – correct solution • 1 – finds $P'(\alpha)$
	(ii)	$P'(\alpha)=0$ $n\alpha^{n-1}+p=0$ $n\alpha^n+p\alpha=0$ $P(x)=x^n+px-q$ $P(\alpha)=0$ $\alpha^n+p\alpha-q=0$ $\alpha^n+p\alpha=q$ Solving simultaneously: $\left(\alpha^n+p\alpha\right)-\left(n\alpha^n+p\alpha\right)=q$ $\alpha^n-n\alpha^n=q$ $\alpha^n(1-n)=q$ $\alpha^n=\frac{q}{1-n}$ $\left(\alpha^{n-1}\right)^n=\left(-\frac{p}{n}\right)^n$ $\left(\alpha^n\right)^{n-1}=\left(\frac{q}{1-n}\right)^{n-1}$ $\left(\frac{q}{1-n}\right)^{n-1}=\left(-\frac{p}{n}\right)^n$ $\left(\frac{-q}{n-1}\right)^{n-1}=(-1)^n\left(\frac{p}{n}\right)^n$ $(-1)^{n-1}\left(\frac{q}{n-1}\right)^{n-1}=(-1)^n\left(\frac{p}{n}\right)^n$ $\left(\frac{q}{n-1}\right)^{n-1}=-\left(\frac{p}{n}\right)^n$ $\left(\frac{p}{n}\right)^n+\left(\frac{q}{n-1}\right)^{n-1}=0$	• 3 – correct solution • 2 – substitutes α , α^n or α^{n-1} into an appropriate expression / equation and attempts to simply • 1 – uses $P(\alpha)=0$ – finds α or α^n
	(iii)	$\alpha=\frac{\alpha^n}{\alpha^{n-1}}$ $=\frac{\left(\frac{q}{1-n}\right)}{\left(-\frac{p}{n}\right)}$ $=\frac{-qn}{p(1-n)}$ $=\frac{qn}{p(n-1)}$	• 1 – correct solution